

Regge gravity from spinfoams

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We consider spinfoam quantum gravity in the double scaling limit $\gamma \rightarrow 0$, $j \rightarrow \infty$ with $\gamma j = \text{const.}$, where γ is the Immirzi parameter, j is the spin and γj gives the physical area in Planck units. We show how in this regime the partition function for a 2-complex takes the form of a path integral over continuous Regge metrics and enforces Einstein equations in the semiclassical regime. The Immirzi parameter must be considered as dynamical in the sense that it runs towards zero when the small wavelengths are integrated out. In addition to quantum corrections which vanish for $\hbar \rightarrow 0$, we find new corrections due to the discreteness of geometric spectra which is controlled by γ .

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INTRODUCTION

Spinfoams [1–3] are a tentative covariant quantization of general relativity. They provide transition amplitudes between quantum states of 3-geometry, in the form of a Misner-Hawking sum over virtual geometries [4, 5]. In the so-called ‘new models’ [6–8], intermediate quantum states are the ones of canonical loop quantum gravity, the $SU(2)$ spin-network states, a remarkable feature that promotes the spinfoam framework to a tentative path integral representation of loop quantum gravity.

The physical picture emerging from the spinfoam gravity is the following: spacetime is a quantum foam of virtual geometries with a discrete and purely combinatorial, relational structure, where the Planck scale plays the role of a natural minimal length. Spinfoams are the result of the quantization of general relativity formulated as a constrained BF theory. In the BF theory with gauge group G , the basic variables are a 2-form B and a connection 1-form ω , both valued in the Lie algebra of G . The equations of motion of BF theory impose the flatness of the connection

$$F(\omega) = d\omega + \omega \wedge \omega = 0 \quad (1)$$

where F is the curvature. So this theory has no local degrees of freedom. The simplicity constraints on the field B turn the topological BF theory into vacuum general relativity, described by the Holst action

$$S = \int (e \wedge e)^* \wedge F(\omega) + \frac{1}{\gamma} \int (e \wedge e) \wedge F(\omega) \quad (2)$$

with gauge group $SO(1,3)$ (or $SO(4)$ in the Euclidean signature). A trace in (2) is understood and $*$ is the Hodge dual in the internal Minkowski (or Euclidean) space. The first term is the Einstein-Hilbert-Palatini action for general relativity in terms of the cotetrad e and the connection ω , regarded as independent variables. The second term vanishes on-shell by using the Cartan structure equation $D_\omega e = 0$, obtained varying ω . Varying e ,

we have the Einstein field equations

$$\epsilon_{IJKL} e^J \wedge F(\omega)^{KL} = 0, \quad (3)$$

equivalent to the Ricci-flatness $R_{\mu\nu} = 0$ of the metric.

The real number $\gamma \neq 0$ is called Barbero-Immirzi, or Immirzi parameter, and controls the spacetime discreteness as it enters the discrete spectra of area and volume operators [9, 10]. Spinfoam models provide a Feynman path integral, or state sum, based on a discretization of (2) over a 2-complex (generalized triangulation) and the full degrees of freedom are recovered in the infinite refinement limit or equivalently summing over 2-complexes [11]. The spinfoam theory is sufficiently simple [12] and possesses the correct symmetries [13]. Recently it has been successfully coupled to matter fields [14, 15]. Furthermore, the large distance analyses were able to extract the correct low-energy physics in some simple cases [16–19].

In spite of those encouraging results, we are still lacking some conceptual steps in the derivation of the covariant form of loop quantum gravity. However, the opposite point of view is to take the quantum theory that has been ‘derived’ in various ways, e.g. in the form [6–8], and check for its physical viability (see section IV of [20] on this viewpoint). A major open problem is to show that the spinfoam Feynman path integral contains Einstein equations in some semiclassical regime. What we would like to have is the spinfoam version of the following semiclassical expansion of the gravitational path integral

$$\int Dg_{\mu\nu} e^{\frac{i}{\hbar} S_{EH}(g_{\mu\nu})} \sim e^{\frac{i}{\hbar} S_{EH}(g_{\mu\nu}^0)} \quad (4)$$

in the classical limit $\hbar \rightarrow 0$, where S_{EH} is the Einstein-Hilbert action for general relativity and on the right hand side it is evaluated on the classical solution $g_{\mu\nu}^0$ of the equations of motion determined by the boundary conditions specified on the metric $g_{\mu\nu}$. Here we propose a solution to this problem.

In a previous paper [21] the authors introduced a simple triangulation of spacetime with three dual vertices

(three 4-simplices) and one internal face (dual to the hinge, where curvature is concentrated) and showed that the spinfoam boundary amplitude is peaked on geometries with nonzero curvature, but the result was limited to the geometry considered and relied strongly on a particular, and somewhat artificial semiclassical limit. In this paper we improve and generalize the previous arguments to a general 2-complex. In the continuous spectrum limit $\gamma \rightarrow 0$ and in the semiclassical limit $\hbar \rightarrow 0$ we find the analogous of the WKB expansion (4) for spinfoam quantum general relativity. The analysis is based on the path integral formulation of references [22, 23].

The paper is organized as follows. First, we review the definition of the EPRL/FK spinfoam model in the Bloch coherent state basis for angular momentum. In the subsequent two sections we perform the continuum and semiclassical limit. After discussing the extension to the Lorentzian signature, we generalize the analysis to the case where a boundary state is present, and motivate an asymptotic formula for the spinfoam transition amplitudes in the semiclassical regime. In the last section we give some outlooks for the future developments.

Throughout the paper we work in natural units $G = \hbar = c = 1$, but will restore some of those constants when needed. In particular, restoring only \hbar the Planck length is $l_P = \sqrt{\hbar}$.

THE SPINFOAM AMPLITUDE

We consider the spinfoam amplitude [6, 8] for a 2-complex σ without matter. We restrict for simplicity to the Euclidean signature and to Barbero-Immirzi parameter $0 < \gamma < 1$, where formulas get simpler. The region of γ considered is not properly a restriction since we are interested in the small γ limit. All the arguments can be easily extended to the Lorentzian signature, that will be discussed later. For each face f (the 2-cells) of the 2-complex σ there is an associated integer spin j_f . Faces are oriented and bounded by a cycle of edges e (the 1-cells). Each edge bounding a face has a source vertex $s(e)$ and a target vertex $t(e)$, where source/target is relative to the orientation of the face. To each edge associate $SU(2)$ elements n_{ef} (f runs over the faces meeting at the edge e) and two source/target

$$Spin(4) \simeq SU(2) \times SU(2) \sim SO(4) \quad (5)$$

gauge group variables $g_{e,s(e)}, g_{e,t(e)}$. The variables n_{ef} can also be interpreted as unit vectors \vec{n}_{ef} in \mathbb{R}^3 , up to a phase ambiguity, by saying that n is a rotation that brings a reference direction to the direction of \vec{n} .

The spinfoam amplitude, or partition function, for the 2-complex σ in the Bloch coherent state basis [24] is de-

fined as

$$W(\sigma) = \sum_{\{j_f\}} \int dg_{ve} \int dn_{ef} \prod_f P_f. \quad (6)$$

The sum is over spins $j_f \in \mathbb{N}$, and the integrals are over the $Spin(4)$ gauge variables and $SU(2)$ variables labeling the edges. We dropped a face normalization factor which is usually taken as the dimension $d(j_f) = 2j_f + 1$ of the $SU(2)$ -irreducible Hilbert space. Other normalizations are possible but irrelevant in the present analysis.

The face amplitude P_f is given by

$$P_f = \text{tr} \overrightarrow{\prod_{e \in f} P_{ef}} \quad (7)$$

where $\overrightarrow{\prod}$ denotes the ordered product of operators (according to the cycle of edges) and

$$P_{ef} = g_{e,s(e)} Y[j_f, n_{ef}] \langle j_f, n_{ef} | Y^\dagger g_{e,t(e)}^{-1}. \quad (8)$$

Here $|j, n\rangle$ is the $SU(2)$ Bloch coherent state [25] for angular momentum¹ along the direction of \vec{n} . The map Y gives the embedding of the $SU(2)$ irreducible representation space with spin j into the $SU(2) \times SU(2)$ representation space with spins (j^+, j^-) , where²

$$j^\pm = \frac{1 \pm \gamma}{2} j. \quad (9)$$

In the canonical basis the embedding map Y is given by the Clebsh-Gordan coefficient

$$\langle j^+ j^-; m^+ m^- | Y | j; m \rangle = C(j^+ j^- j; m^+ m^- m). \quad (10)$$

Using the decomposition of the $SO(4)$ variables g in self-dual and antiselfdual rotations $(g^+, g^-) \in SU(2) \times SU(2)$ and the factorization properties of $SU(2)$ coherent states [26] we can write

$$P_{ef} = P_{ef}^+ \otimes P_{ef}^- \quad (11)$$

with

$$P_f^\pm = \overrightarrow{\prod_{e \in f} P_{ef}^\pm} \quad (12)$$

and

$$P_{ef}^\pm = g_{e,s(e)}^\pm [n_{ef}]^{\otimes 2j_f^\pm} \langle n_{ef} |^{\otimes 2j_f^\pm} (g_{e,t(e)}^\pm)^{-1} \quad (13)$$

¹ We introduced the notation $|j, n\rangle$ for the standard antilinear map ϵ applied to $|j, n\rangle$. In the standard basis, the one that diagonalizes J^2 and J_z , it is given by the symbol $^j \epsilon_{mm'} = (-1)^{j+m} \delta_{m,-m'}$.

² Because of equation (9) γ is quantized to be rational. This restriction is not present in the Lorentzian version of the theory.

where the Bloch coherent states are now in the fundamental $j = \frac{1}{2}$ representation. Define also the face amplitudes in the fundamental representation as

$$P_f^\pm \Big|_{j_f=\frac{1}{2}} = \tilde{P}_f^\pm, \quad (14)$$

a notation will be useful later. The partition function (6) can then be written in the form of a path integral

$$W(\sigma) = \sum_{\{j_f\}} \int dg_{ev} \int dn_{ef} e^S \quad (15)$$

with complex action given by

$$S = \sum_f S_f = \sum_f \ln P_f. \quad (16)$$

This is completely equivalent to (6). If we want to focus on the nondegenerate part of the state sum, we can eliminate a small integration region in the neighborhood of a zero measure set defined as follows

THE CONTINUUM $\gamma \rightarrow 0$ LIMIT

The continuum limit of the theory is defined as the infinite refinement of the 2-complex [11] (possibly undergoing a second order phase transition to a smooth spacetime), where we expect a large portion of spacetime to be described as the union of many small elementary regions (quanta) labeled by spins of order one. Differently, here we define a continuum limit that is suitable to describe gravitational physics with a truncation of the theory on a finite cellular structure of spacetime, namely with an approximation of the full theory. The spinfoam amplitudes associated to finite graphs can be viewed as effective amplitudes obtained after a coarse-graining procedure [27, 28] applied to the complete theory. But since the truncated amplitudes are not fundamental, there is no reason to keep fixed the Immirzi parameter to its ‘bare’ value γ_0 . The possibility of a renormalization of the Barbero-Immirzi parameter has been recently advocated in different contexts [29, 30].

Here we explore the possibility of a running towards zero, $\gamma \rightarrow 0$, simultaneously with the large spin regime of the theory. Thus we consider

$$j \rightarrow \infty, \gamma \rightarrow 0, j\gamma = \text{const.} \quad (17)$$

and γj is the macroscopic physical area in Planck area units. Notice that the Immirzi parameter controls the spacetime discreteness. In particular it controls the area gap and the spacing between area eigenvalues, thus the limit (17) is the continuum limit for the area operator. The correctness of this approximation has to be checked against concrete computations in specific examples, or possibly be justified and derived from the full amplitude

(defined on the infinite 2-complex) as the result of the iteration of some kind of renormalization group transformation at the level of the spinfoam ‘lattice’. The latter possibility is intriguing but, to our knowledge, a little far away from the present computational techniques.

In order to see the effect of (17) on the partition function, let us restrict our attention to a 2-complex which is dual to a simplicial triangulation. Now every vertex is surrounded by five edges and ten faces, and every edge bounds four faces. Vertices are dual to 4-simplices, edges are dual to tetrahedra, and faces are dual to triangles. The analysis parallels the one of [23, 31] at fixed, large spins, and will exclude the degenerate part of the state sum, as done in [23].

We are interested in making explicit the dependence on the Immirzi parameter, so let us decompose the action (16) using (9) in the following way

$$S_f = \gamma j_f (\ln \tilde{P}_f^+ - \ln \tilde{P}_f^-) + j_f (\ln \tilde{P}_f^+ + \ln \tilde{P}_f^-) \quad (18)$$

or, equivalently,

$$S_f = a_f (\ln \tilde{P}_f^+ - \ln \tilde{P}_f^-) + \frac{1}{\gamma} a_f (\ln \tilde{P}_f^+ + \ln \tilde{P}_f^-) \quad (19)$$

where we have defined the area of the triangle dual to f as

$$a_f = \gamma j_f, \quad (20)$$

according to the area spectrum of loop quantum gravity.

We are interested in the evaluation of the amplitude (15) in a region of macroscopic areas a_f and in the limit $\gamma \rightarrow 0$ with a_f fixed (so $j_f \rightarrow \infty$). Collecting all the face terms (19), let us write the full action as

$$S = S^0 + \frac{1}{\gamma} S' \quad (21)$$

and the path integral

$$W(\sigma) = \sum_{\{a_f\}} \int dg_{ve} \int dn_{ef} e^{S^0 + \frac{1}{\gamma} S'} \quad (22)$$

where now the action S is viewed as a function of the areas, and consequently the summation is over γ -multiples of the integer spins. The continuum approximation of this sum can be taken only if the sum is dominated by a region of areas that contains a sufficiently large number of area eigenvalues. This condition can be satisfied only when we consider the more general, and more physical, transition amplitudes associated to a 2-complex with boundary. This will be discussed later. For the moment, let us suppose we can do this approximation and write

$$W(\sigma) \simeq \int da_f \int dg_{ev} \int dn_{ef} e^{S^0 + \frac{1}{\gamma} S'} \quad (23)$$

where we have dropped³ one global factor γ^{-1} per each face, that takes into account the measure of the small area intervals in the Riemann sum.

For the stationary phase evaluation of (23) we will have to take variations of the second term S' of the full action (21), because it is the one proportional to the large parameter $\frac{1}{\gamma} \rightarrow \infty$. Being S' complex, in addition to stationary points we have also to find out the points that maximize the real part. The action S' has nonpositive real part $\text{Re } S' \leq 0$, so the main contribution to the integral over a_f , g_{ev} and n_{ef} comes from the critical points where

$$\text{Re } S' = 0. \quad (24)$$

Since both P_f^+ and P_f^- have nonpositive real part, the condition (24) holds for $\text{Re } P_f^+ = \text{Re } P_f^- = 0$. Using that the scalar product of coherent states satisfies $|\langle n_1 | n_2 \rangle| = \frac{1 + \vec{n}_1 \cdot \vec{n}_2}{2}$, we get easily that the critical points are the solutions to the following system

$$g_{ev}^+ \vec{n}_{ef} = -g_{e'v}^+ \vec{n}_{e'f}, \quad (25)$$

$$g_{ev}^- \vec{n}_{ef} = -g_{e'v}^- \vec{n}_{e'f}, \quad (26)$$

where e, e' are adjacent edges in the face f , sharing the vertex v . In the last equations g^\pm acts in the vector representation on \mathbb{R}^3 . If there are no solutions, the amplitude is exponentially suppressed. Using (25, 26) we have also that the brackets

$$\langle n_{ef} | (g_{ve}^\pm)^{-1} g_{ve'}^\pm | n_{e'f} \rangle = e^{i\theta_{vf}^\pm} \quad (27)$$

reduce to simple phases, on the critical points (on-shell).

Furthermore, we must require that on the critical points the action is stationary. Varying S' with respect to $SO(4)$ group variables, and evaluating at a critical point we obtain the condition

$$\delta_{g_{ev}} S' |_{crit.} = 0 \longrightarrow \sum_{f \in e} a_f \vec{n}_{ef} = 0 \quad (28)$$

which expresses the closure relation for the tetrahedron dual to the edge e . Variation with respect to the unit vectors n_{ef} does not give further information, because it is automatically satisfied. Before completing the continuum approximation, let us open a short discussion on the geometrical interpretation of the critical points.

The existence of critical points is related to the existence of a triangulation where the areas⁴ of triangles are

specified by the set a_f , and the unit normals to the triangles (viewed in the 3-dimensional space of each tetrahedron e), are specified by the set of unit vectors n_{ef} . Thus the critical points have the nice interpretation of a 4-dimensional Regge manifold, that is a manifold endowed with a continuous, piecewise flat metric where curvature is distributional and concentrated on triangles. This can be seen as follows. First, define the unit vectors

$$\vec{n}_{vf}^\pm = g_{ev}^\pm \vec{n}_{ef} \quad (29)$$

where g_{ev}^\pm and \vec{n}_{ef} are a solution of the critical system (26) and the closure relation. Then the vectors

$$\vec{J}_{vf}^\pm = j_f^\pm \vec{n}_{vf}^\pm \quad (30)$$

can be interpreted as the selfdual (+) and antiselfdual (−) components of the Lie algebra valued discrete field

$$J_{vf} = B_{vf} + \frac{1}{\gamma} B_{vf}^* \in so(4) \quad (31)$$

from which we can extract the field B_{vf} . The field B_{vf} codes the spacetime metric degrees of freedom because

$$A_{vf} = \frac{8\pi G\hbar}{c^3} B_{vf}^* = E_{vfb_1} \wedge E_{vfb_2} \quad (32)$$

is the simple area bivector⁵ of the triangle f , in the frame of the 4-simplex v , where we have restored dimensional units. It is the simplicial version of $e \wedge e$, and it is constructed via the vectors $E_{vfb} \in \mathbb{R}^4$ along two sides, labeled with b , of the triangle dual to f , in a flat coordinate patch covering the 4-simplex v .

So far we have reconstructed the geometry of the 4-simplices out of a given solution (a_f, g_{ev}, n_{ef}) of the critical system (25, 26) and the closure relation (28) (see [31] for the 4-simplex reconstruction theorem). The next step is to check if 4-simplices are consistently glued together, by identification of the shared tetrahedra, so that they form a proper spacetime triangulation, and (32) provides a consistent discrete tetrad over the simplicial manifold. By construction, the Euclidean geometry of the tetrahedron dual to e is the same when viewed from the source 4-simplex $s(e)$ or from the target 4-simplex $t(e)$.

However there is a discrete ambiguity in the way 4-simplices are glued [35]. Indeed from any given solution (a_f, g_{ev}, n_{ef}) we can generate the finite set of parity-related solutions by a flip in the selfdual and antiselfdual group variables g_{ev}^+, g_{ev}^- in an arbitrary subset of vertices v . This is equivalent to a parity transformation, or reorientation, of a subset of 4-simplices. Thus (a_f, g_{ev}, n_{ef})

³ We are only interested in the oscillatory behaviour of the integral.

⁴ It is well known that a generic assignment of areas does not correspond to any Regge triangulation [32–34]. In other words, in general there exists no assignment of lengths l_b for the sides b of triangles such that $a_f = a_f(l_b)$. The critical equations select only the Regge-like area configurations.

⁵ We speak about bivectors or elements in the Lie algebra as if they were the same thing, using the algebra isomorphism $\Lambda^2 \mathbb{R}^4 \simeq Lie(O(4))$.

labels a class of solutions, and the elements of this class are specified with the set ϵ_v of orientations of 4-simplices. Now we have that two 4-simplices are glued consistently with a Regge triangulation only if their orientations induce opposite orientations in the common tetrahedron. If the last condition is true for every couple of vertices, the solution has the correct global orientation and describes a Regge manifold. In all the other cases it describes only a generalized Regge manifold, with no simple geometrical interpretation, and will produce a generalized Regge action, as we shall see.

The spin connection is coded in the $SO(4)$ holonomy U_e that parallel transports the bivectors from one 4-simplex to another along the edge e . It is the unique solution of the simplicial Cartan structure equation

$$E_{t(e),fb} = U_e E_{s(e),fb} \quad (33)$$

and defined⁶ in terms of the on-shell gauge variables in a simple way:

$$U_e = g_{e,t(e)} g_{e,s(e)}^{-1}. \quad (35)$$

Observe that (27) implies that the full holonomy around the loop (boundary of the face) has the form

$$U_f = \overrightarrow{\prod}_{e \in f} U_e = e^{\Theta_f B_{vf} + \Theta_f^* B_{vf}^*}, \quad (36)$$

where the first holonomy in the product is along the edge e such that $s(e) = v$, so the holonomy is based at v . Thus U_f is a $SO(4)$ rotation that decomposes into a rotation along the plane of B_{vf} , and a rotation along its dual orthogonal plane, the one of B_{vf}^* , and preserves both B_{vf} and B_{vf}^* (bivectors have no definite location in their plane). The angles are given respectively by

$$\Theta_f = \sum_{v \in f} (\theta_{vf}^+ - \theta_{vf}^-), \quad (37)$$

$$\Theta_f^* = \sum_{v \in f} (\theta_{vf}^+ + \theta_{vf}^-). \quad (38)$$

However, the Cartan structure equation (33) puts a further constraint on U_f . Apart from preserving B_{vf}^* , it must preserve vectors in the plane of B_{vf}^* . It follows that

$$A_{t(e),f} = U_e A_{s(e),f}. \quad (34)$$

Considering a single face, the previous equation still does not determine whether it parallel transports the side vectors $E_{s(e),fb}$ as well, because we can do an arbitrary rotation of the target $E_{t(e),fb}$ along the plane of $A_{t(e),f}$ without affecting the condition (34). However (34) holds for any of the four faces bounded by e . It follows that U_e parallel transports the side vectors as well, thus U_e is the spin connection.

Θ_f^* vanishes. Interestingly, Θ_f^* is an angle with the interpretation of a torsion degree of freedom and accordingly it must vanish since the on-shell discrete connection U_e is torsion-free (see also [23] for more details on torsion).

It is finally time to get back to the small γ expansion. The last thing to do is to take variations of S' with respect to the areas a_f , but this does not give further restrictions, because

$$\left. \frac{\partial S'}{\partial a_f} \right|_{crit.} = \frac{i}{\gamma} \Theta_f^* = 0 \quad (39)$$

is automatically satisfied on the critical points. Thus in the $\gamma \rightarrow 0$ limit the total action S must be evaluated at a critical point of S' , and using (27) we have

$$S|_{crit.} = S_R = i \sum_f a_f \Theta_f. \quad (40)$$

The angle Θ_f is the generalized deficit angle, given by (2π minus) the sum of all the 4-dimensional dihedral angles $\theta_{vf}^+ - \theta_{vf}^-$ between the neighboring tetrahedra that share the triangle dual to the face f . Notice that if the solution is such that the 4-simplices around a face are glued with consistent orientations then Θ_f is the usual deficit angle of Regge calculus.

The deficit angle codes the spacetime curvature concentrated on the triangle. Its relation with the holonomy around the face is given by the previous formula (36) with $\Theta_f^* = 0$.

The quantity (40) is the Regge form [36, 37] of the action S_R for general relativity. However here what we get is a generalized Regge action, in the sense of Barrett and Foxon [38], a situation similar to the Ponzano-Regge model for spinfoam gravity in three spacetime dimensions [39]. The exponential of (40) is the contribution to the partition function associated to a given critical point. In order to write the asymptotic approximation of the integral (23) we have to sum over all critical points. In general this will take the form of a continuous sum over the set \mathcal{C} of critical points (critical manifold), with some measure μ that in principle can be computed by standard asymptotic analysis tools, in the following way.

Suppose we are given an n -dimensional integral of the form

$$I = \int dx a(x) e^{\lambda f(x)} \quad (41)$$

and we want to compute its asymptotic expansion for large positive λ . Suppose the action f has nonpositive real part and a continuous set \mathcal{C} (hypersurface) of critical points $y \in \mathcal{C}$ where $\text{Re } f(y) = 0$ and $\partial f / \partial x^i|_y = 0$. Then we have the following expansion:

$$I \sim e^{\lambda f(y_0)} \int_{\mathcal{C}} dy \mu(y) \frac{a(y)}{\sqrt{\det H^\perp(y)}} (1 + \mathcal{O}(1/\lambda)) \quad (42)$$

⁶ From its definition, it is evident that U_e parallel transports the area bivectors

where H^\perp is the Hessian matrix of f restricted to the directions normal to the critical surface with respect to some metric, and μ is the measure induced on the critical surface by the same metric. In the factor on the left of the integral sign, the action is evaluated on an arbitrary reference critical point y_0 (but it is of course independent of this choice).

Using the general formula (42) for $f = S'$, $a = \exp(S^0)$, and $\lambda = 1/\gamma$ we find for the partition function

$$W(\sigma) = \int_{\mathcal{C}} d\mu(a_f, n_{ef}, g_{ev}) e^{iS_R} + \mathcal{O}(\gamma) \quad (43)$$

where $\mathcal{O}(\gamma)$ denotes the γ -corrections to the amplitude that come from the next-to-leading orders of the asymptotic approximation.⁷ The integration region \mathcal{C} contains also the contributions from all the possible orientations ϵ_v of 4-simplices.

THE SEMICLASSICAL $\hbar \rightarrow 0$ LIMIT

More interestingly, let us parametrize the previous integration (43) using length variables. Given that a class of parity-related critical points (a_f, n_{ef}, g_{ev}) corresponds to a Regge triangulation⁸, there exists an assignment of lengths l_b to the sides b of the triangles such that the areas a_f coincide with the areas computed out of the lengths l_b :

$$a_f = a_f(l_b). \quad (44)$$

Exploiting this fact, we can parametrize the critical manifold \mathcal{C} with the set of side lengths l_b and with the joint orientations $\epsilon = (\epsilon_1, \epsilon_2, \dots)$ of the 4-simplices. Restoring the \hbar dependence, we rewrite (43) as

$$W(\sigma) = \sum_{\epsilon} \int d\tilde{\mu}(l_b) e^{\frac{i}{\hbar} S_R^\epsilon(l_b)} + \mathcal{O}(\gamma), \quad (45)$$

and the generalized Regge action is now an explicit function of the lengths

$$S_R^\epsilon(l_b) = \sum_f a_f(l_b) \Theta_f^\epsilon(l_b) \quad (46)$$

as in the original classical formulation [36]. We have also made the sum over the orientations ϵ explicit, so the generalized deficit angle at a face depends also on the particular orientation of the 4-simplices around the face.

The last expressions are a good starting point for taking the semiclassical limit. Before discussing this, let us make a few comments on the effect of sending the Immirzi parameter to zero. The remarkable consequence of the continuum limit $\gamma \rightarrow 0$ is that the spinfoam amplitude reduces effectively, that is up to γ -corrections, to a quantization of Regge gravity [40] given by formula (45) where the fundamental variables are continuous lengths, the major difference being the nonzero contribution of the non-geometric orientations. The result resonates with the recent findings in the computation of the graviton propagator within loop quantum gravity [16–18]. In particular, as shown in [18], the leading order (in the \hbar expansion) graviton propagator $G(x, y)$ presents the same kind of γ -corrections,

$$G(x, y) = \frac{R + \gamma X + \gamma^2 Y}{|x - y|^2} + \hbar\text{-corr.} \quad (47)$$

and only in the limit $\gamma \rightarrow 0$ the tensorial structure of the 2-point function matches with the matrix of correlations R computed in quantum Regge gravity, and, even more interestingly, with the one given by standard perturbative gravity on flat space. In retrospect, at the light of the present general analysis the previous result (47) is much more clear.

Suppose now we are interested in the semiclassical expansion of the partition function (45). This corresponds to looking at a region of the integration domain where the areas are macroscopic, that is large as compared to the Planck area,

$$\frac{a_f}{l_P^2} \gg 1, \quad (48)$$

or equivalently to the standard formal WKB expansion $\hbar \rightarrow 0$. Remember that we are still in the small Immirzi parameter regime. We can state the regime in a more suggestive fashion by introducing two physical scales. One is the length scale of *quantum gravity*, identified with the Planck length

$$l_{QG} \equiv l_P, \quad (49)$$

the other is the scale of *loop quantum geometry*, that is the scale where we can ‘see’ the discreteness of spacetime

$$l_{QG} \equiv \sqrt{\gamma} l_P. \quad (50)$$

Thus the regime of the spinfoam path-integral we are looking for is expressed by the following relation:

$$l \gg l_{QG} \gg l_{LQG} \quad (51)$$

where l is the typical linear scale of each 4-simplex. This regime can be selected by appropriate semiclassical boundary conditions in the transition amplitudes, as explained later.

⁷ The corrections in γ should contain also the error we make by approximating the sums with integrals. This is expected to decay at least exponentially.

⁸ Only the consistently oriented elements of this class have to be interpreted as a proper Regge triangulation.

The classical equations of motion are obtained by varying (46) with respect to the lengths. Using also the Schlafli identity [36], which tells that the variation of the deficit angles does not contribute to the total variation of the action, these are the well known Regge equations

$$\sum_f \frac{\partial a_f}{\partial l_b} \Theta_f^\epsilon = 0, \quad (52)$$

a discrete version of the continuum Einstein equations in vacuum

$$R_{\mu\nu} = 0, \quad (53)$$

that is of the vanishing of the Ricci tensor. Notice that the interpretation of (52) as Einstein equations is clearer (probably possible only) when ϵ is a consistent global orientation.

The Regge equations (52) enforce a relation between the generalized deficit angles of different faces. The integral (45) is dominated by its stationary ‘trajectories’, namely by the sets of lengths l_b which are a solution of the Regge equations (52). However, in order to pick up a single classical trajectory we need to specify appropriately the boundary conditions, or in other words we have to consider the transition amplitudes. The spin-foam boundary formalism for the transition amplitudes is briefly reviewed after the following Lorentzian section.

LORENTZIAN SIGNATURE

The model in its Lorentian version [7] has pretty much the same form as the Euclidean one, *mutatis mutandis*. The gauge group is now

$$SL(2, \mathbb{C}) \sim SO(1, 3). \quad (54)$$

As before, the partition function is

$$W(\sigma) = \sum_{\{j_f\}} \int dg_{ve} \int dn_{ef} \prod_f P_f, \quad (55)$$

with face amplitude P_f given by the trace of operators

$$P_f = \text{tr} \prod_{e \in f} P_{ef} \quad (56)$$

and where the elementary edge operator is the infinite dimensional matrix

$$P_{ef} = g_{e,s(e)} Y[j_f, n_{ef}] \langle j_f, n_{ef} | Y^\dagger g_{e,t(e)}^\dagger. \quad (57)$$

Let us explain the previous formulas. As in the Euclidean theory, we have the same $SU(2)$ spins j_f and Bloch $SU(2)$ coherent states labeled by n_{ef} . The gauge group variables g_{ev} are elements in $SL(2, \mathbb{C})$, integrated with the Haar measure, and the redundant integrations

(infinite volumes of $SL(2, \mathbb{C})$) have to be dropped in order to make the amplitude potentially finite [41]. The map Y is constructed through the principal series of unitary representations $\mathcal{H}^{(k,\rho)}$ of $SL(2, \mathbb{C})$, which are infinite dimensional and labeled by an integer k and a positive real number ρ . We have the useful orthogonal decomposition

$$\mathcal{H}^{(j,\rho)} = \bigoplus_{k \geq j} \mathcal{H}_k^{(j,\rho)} \quad (58)$$

where the subspace $\mathcal{H}_j^{(k,\rho)}$ carries a representation of spin j of the $SU(2)$ subgroup of $SL(2, \mathbb{C})$ rotations that leave a fixed timelike 4-vector invariant. The map Y in (57) is defined in terms of this decomposition as the isometric embedding of $SU(2)$ irreducible representations j in the lowest weight term in (58), namely $k = j$. Moreover, the Lorentzian model imposes the following constraint on the labels k and ρ :

$$\rho = \gamma k, \quad (59)$$

which is the analogous of (9). Notice that the previous equation does not impose a quantization of the Immirzi parameter, that can be any positive number, a comforting feature from the perspective of the contact with Hamiltonian loop quantum gravity, where γ is not quantized as well. Hence the map Y is the embedding of the $SU(2)$ representations j into the $SL(2, \mathbb{C})$ representations $(j, \gamma j)$. This completes the definition of the Lorentzian model.

In order to write a path integral form of the model, it is useful to realize the Hilbert space $\mathcal{H}^{(k,\rho)}$ in the usual way as the space of homogeneous functions of $z \in \mathbb{C}^2$ with a specific measure Ω that defines the scalar product

$$(f, g) = \int_{\mathbb{CP}^1} dz \Omega(z) \overline{f(z)} g(z) \quad (60)$$

and the integration on the complex projective line \mathbb{CP}^1 makes sense because Ω is such that the integrand 1-form is invariant under rescaling of z . Thus $z = (z_1, z_2) \in \mathbb{C}^2$ can be interpreted as homogeneous coordinates $[z_1 : z_2]$ for \mathbb{CP}^1 . It is a simple exercise in representation theory to compute explicitly the integral kernel of the edge operator (57), which has the following expression

$$P_{ef}(z, z') = \Omega_{ef} \left(\frac{\langle z | g_{e,s(e)} [n_{ef}] \langle n_{ef} | g_{e,t(e)}^\dagger | z' \rangle}{|g_{e,s(e)}^\dagger z|^{1-i\gamma} |g_{e,t(e)}^\dagger z'|^{1+i\gamma}} \right)^{j_f} \quad (61)$$

where the function Ω_{ef} is slowly varying. Now we define the analogous of (19) as

$$S_{ef} = ia_f \log \tilde{P}_{ef}^0 + \frac{a_f}{\gamma} \log \tilde{P}'_{ef} \quad (62)$$

with

$$\tilde{P}_{ef}^0 = \frac{|g_{e,s(e)}^\dagger z_{s(e),f}|}{|g_{e,t(e)}^\dagger z_{t(e),f}|} \quad (63)$$

$$\tilde{P}_{ef}' = \frac{\langle z_{s(e),f} | g_{e,s(e)} [n_{ef}] \langle n_{ef} | g_{e,t(e)}^\dagger | z_{t(e),f} \rangle}{|g_{e,s(e)}^\dagger z_{s(e),f}| |g_{e,t(e)}^\dagger z_{t(e),f}|} \quad (64)$$

and the total action for the 2-complex σ as

$$S = \sum_f \sum_{e \in f} S_{vf} = S^0 + \frac{1}{\gamma} S' \quad (65)$$

where in the last equality we have collected all the terms proportional to $\frac{1}{\gamma}$. From the form of the edge kernel (61) it is clear that this action formulation is equivalent to the original definition of the partition function (55), and our final form is

$$W(\sigma) = \sum_{\{a_f\}} \int dg_{ev} \int dn_{ef} \int dz_{vf} \omega e^S \quad (66)$$

with measure density given by

$$\omega = \prod_f \prod_{e \in f} \Omega_{ef}. \quad (67)$$

Observe that we have one integration dz_{vf} on \mathbb{CP}^1 per each couple (v, f) of the 2-complex. The action (65) and the variables used are a simple generalization to an arbitrary 2-complex σ of the single vertex action of [42]. Notice also that S^0 is purely imaginary while S' is complex. While the previous definitions are general, from now on we restrict our attention to the particular case of a simplicial 2-complex.

In the continuum limit we look for the critical points of the complex action S' . Imposing $\text{Re } S' = 0$ and the vanishing of variations of S' with respect to z_{vf} we find two equations

$$g_{ev} [n_{ef}] = \frac{|g_{e'v}^\dagger z_{vf}|}{|g_{e'v}^\dagger z_{vf}|} g_{e'v} [n_{e'f}] \times \text{phase} \quad (68)$$

$$g_{ev}^{\dagger-1} [n_{ef}] = \frac{|g_{e'v}^\dagger z_{vf}|}{|g_{e'v}^\dagger z_{vf}|} g_{e'v}^{\dagger-1} [n_{e'f}] \times \text{phase} \quad (69)$$

for the critical points, where e and e' are two adjacent edges in the boundary of the face f , both bounded by the vertex v . The requirement of stationarity of S' with respect to the n_{ef} variables gives the trivial identity $\vec{n}_{ef} - \vec{n}_{e'f} = 0$. The variation of S' with respect to the areas a_f is automatically zero on the critical points, because of the torsionless condition, similarly to the Euclidean case. Varying with respect to g_{ev} and using the previous critical equations we get the closure condition in each tetrahedron:

$$\delta_{g_{ev}} S'|_{crit.} = 0 \longrightarrow \sum_{f \in e} a_f \vec{n}_{ef} = 0. \quad (70)$$

The solutions to those equations can be interpreted as 4-dimensional Lorentzian Regge manifolds, using the following construction. Given a solution, consider g_{ev} , n_{ef} and out of them define two null (lightlike) 4-vectors in Minkowski space

$$N_{vf} = g_{ev}(1, \vec{n}_{ef}), \quad (71)$$

$$\tilde{N}_{vf} = g_{ev}(1, -\vec{n}_{ef}). \quad (72)$$

Taking their wedge product, define the spacelike simple area bivectors⁹

$$A_{vf} = a_f (N_{vf} \wedge \tilde{N}_{vf})^* \in so(1, 3) \quad (73)$$

that code the geometry of a curved spacetime triangulated with flat 4-simplices. Indeed for each vertex v , these bivectors describe completely the geometry of a Lorentzian flat 4-simplex with spacelike boundary: there exists a Lorentzian 4-simplex with area bivectors defined as (73), whereas in the case the variables are not a solution of the critical equations such a 4-simplex does not exist (see [42], [43] on the reconstruction theorems for the 4-simplex). Two neighboring 4-simplices are described by two sets of ten bivectors $A_{s(e),f}$ and $A_{t(e),f}$. The 4-simplices are glued together and share the tetrahedron dual to e . The 3-dimensional Euclidean geometry of the common tetrahedron is the same when viewed from $s(e)$ or $t(e)$ by construction of the model; this is due to the fact that in the edge amplitude (57) the source and target unit vectors are not independent but are both equal to n_{ef} . However, as in the Euclidean signature we can glue two 4-simplices in two ways, one with the correct orientation, the other with the ‘wrong’ orientation, and both solutions are allowed.

Thus (73) defines the metric of a (generalized) Lorentzian Regge manifold, with Levi-Civita connection coded in the parallel transport

$$U_e = g_{e,t(e)} g_{e,s(e)}^{-1}, \quad (74)$$

interpreted as the path-ordered exponential of the Lorentz connection from one 4-simplex to another. The boundary of every 4-simplex is formed by five spacelike tetrahedra. It can be shown [42] that the real ratio

$$\frac{|g_{e'v}^\dagger z_{vf}|}{|g_{ev}^\dagger z_{vf}|} \equiv \exp \theta_{vf} \quad (75)$$

of equations (68), (69) yields the Lorentzian dihedral ‘angle’ θ_{vf} between the two tetrahedra dual to e and e' in the boundary of the 4-simplex v . It is not an angle, but rather a boost parameter. Its hyperbolic cosine is the

⁹ We always trade bivectors for Lie algebra elements and viceversa, using the isomorphism $\Lambda^2 \mathbb{R}^{1,3} \simeq Lie(O(1,3))$.

scalar product of the unit timelike outward normals to the tetrahedra, computed in the 4-simplex v , up to a sign. If one normal is future pointing and the other past pointing the dihedral angle is defined to be positive. If both are future pointing, or past pointing, the angle is defined negative.

The second term S' vanishes on the critical points, because of the torsionless condition (analogous to the Euclidean case). So the total action gives the on-shell contribution

$$S|_{crit.} = S_R = i \sum_f a_f \Theta_f \quad (76)$$

where we have defined

$$\Theta_f = \sum_{v \in f} \theta_{vf} \quad (77)$$

which is the generalized Lorentzian deficit angle at the face f , dual to a spacelike triangle. Notice that similarly to the Euclidean signature, also in a Lorentzian triangulation the deficit angle is zero for a flat connection. The on-shell action (76) is the (generalized) Regge action for Lorentzian gravity. It is worth observing again that the critical equations admit solutions only for Regge-like configurations of the areas, because we cannot reconstruct the geometry (73) using areas which are not admissible. From here, the analysis of the continuum and semiclassical limits proceeds exactly as in the case of Euclidean signature.

GENERAL BOUNDARY AMPLITUDES

The predictions of the spinfoam theory are the transition amplitudes. These are associated to general boundaries of the 2-complexes [44]. Given a 2-complex with boundary (the boundary is interpreted to be spacelike in the Lorentzian theory), its boundary graph Γ is an abstract oriented graph made of links l (where the external faces end) and nodes n (where the external edges end). The boundary graph inherits its labeling and orientation from the external faces and edges. The set formed by the boundary graph Γ , the spins j_l associated to the links, and unit vectors n_{nl} associated to the nodes is the boundary data. Notice that for a simplicial 2-complex the boundary graph contains only 4-valent nodes. So for each node we have exactly four labels n_{nl} , where l labels the four links bounded by the node n . These are interpreted as the four normals to the faces of the tetrahedron dual to the node.

The spinfoam transition amplitude for the 2-complex σ with boundary Γ in the Bloch coherent state basis [24] is a functional of the boundary data given by the following

expression

$$W(j_l, n_{ne}) = \sum_{\{j_f\}} \int dg_{ve} \int dn_{ef} \prod_l P_l \prod_f P_f, \quad (78)$$

where now the face amplitude is split in a boundary (relative to external faces l), and a bulk part (relative to internal faces f), and accordingly the two products are over the external and internal faces respectively. Let us explain better the last formula. First, the external face amplitude P_l is the same as (7) except that for the edges ending on the boundary we have only ‘half’ of (8), namely

$$P_{ef} = \langle j_f, n_{ef} | Y^\dagger g_{e,t(e)}^\dagger \quad (79)$$

or

$$P_{ef} = g_{e,s(e)} Y[j_f, n_{ef}] \quad (80)$$

depending on the orientation of the edge induced by the face orientation. Second, the summation is over the sole internal spins $j_f \in \mathbb{N}$, and the integrals are over the $Spin(4)$ gauge variables in the Euclidean, or $SL(2, \mathbb{C})$ in the Lorentzian, and also over $SU(2)$ variables (the unit vectors) labeling the internal edges.

The amplitude (78) for a 2-complex with boundary has the nice interpretation as transition amplitude associated to a spin-network supported on the boundary graph Γ . The canonical loop quantum gravity Hilbert space associated to the graph Γ is

$$\mathcal{H}_\Gamma = L^2(SU(2)^L / SU(2)^N) \quad (81)$$

with L the number of links and N the number of nodes. If Γ has two components $\Gamma = \Gamma_{in} \cup \Gamma_{out}$ we could interpret the transition amplitude (78) as a standard transition from an initial state in $\mathcal{H}_{\Gamma_{in}}$ to a final state in $\mathcal{H}_{\Gamma_{out}}$. However it could be more correct to call those amplitudes *extraction amplitudes*, because in the spinfoam paradigm they are supposed to ‘extract’ the physical states by projecting the kinematical boundary states down to the Hilbert space of the solutions of the Hamiltonian constraint.

In the state space (81) there exists an overcomplete basis of spin-networks, the Bloch coherent state basis introduced in quantum gravity by Livine and Speziale [24] labeled precisely by j_l and n_{nl} . Thus (78) can be interpreted as the covariant transition amplitude for the quantum states of 3-geometry given by the loop quantum gravity states $|j_l, n_{nl}\rangle$,

$$W(j_l, n_{ne}) = \langle W | j_l, n_{nl} \rangle. \quad (82)$$

The semiclassical analysis of the previous section is straightforwardly generalized to a 2-complex with boundary, namely to the transition amplitude (78), once we

write it in the path integral form. For example, in the Lorentzian case we rewrite

$$W(j_l, n_{ne}) = \sum_{\{j_f\}} \int dg_{ve} \int dn_{ef} \int dz_{vf} \omega e^{\sum S_l + S_f}. \quad (83)$$

The boundary formalism allows to select the continuum/semiclassical regime in the following way. First, with a little abuse of notation, write the boundary functional in terms of the areas as

$$W(a_l, n_{ne}). \quad (84)$$

Then we look at the behavior of this transition amplitude when all the boundary areas a_l are macroscopic, in the limit $\gamma \rightarrow 0$ with a_l fixed. This is the

$$a \gg l_{QG}^2 \gg l_{LQG}^2 \quad (85)$$

regime. It is formally the simultaneous semiclassical limit $\hbar \rightarrow 0$ and continuum limit $\gamma \rightarrow 0$, and corresponds to looking at macroscopic geometries and at the same time neglecting the quantum discreteness of spacetime¹⁰. Suppose also that the path integral is dominated by areas of the same order of magnitude of the boundary areas: $a_l \simeq a_f$. In other words, the macroscopic boundary state must enforce macroscopic areas in the bulk of the triangulation, a condition that can be checked for the specific geometry chosen.

If we go through the previous analysis of the partition function, it becomes clear that the boundary amplitude receives the dominant contribution from the solutions of the generalized Regge equations (52) compatible with the specified boundary data (see [45, 46] on the initial value problem). The nature and the number of solutions has to be studied in detail case by case. As an example, in the case there is only one solution of the Regge equations the behaviour of the boundary amplitude is¹¹

$$W(a_l, n_{ne}) \sim A e^{\frac{i}{\hbar} S_R(a_l, n_{ne})} + B e^{-\frac{i}{\hbar} S_R(a_l, n_{ne})} \quad (86)$$

where now S_R is the Hamilton function, that is the action evaluated at the classical trajectory determined by the boundary data via Einstein equations. The two exponentials with reversed global sign are related to the two opposite global orientations of the triangulation, and the quantum theory sums over them with a different weight, even if classically they represent the same un-oriented spacetime. This result for general triangulations is similar to the single vertex results of [31, 42, 47, 48]. In

general we expect non-zero contributions also from the non-geometric orientations that must be added to (86); this may happen if there are solutions of the generalized Regge equations with the same boundary data.

Notice that the boundary data a_f, n_{nl} may fail to be a consistent labeling of the boundary spin-network, namely there may not exist a 3-dimensional (spacelike, in the Lorentzian case) triangulation with the prescribed areas and unit vectors, the latter interpreted as the unit normals to the faces of the tetrahedra. Due to the geometric interpretation of the critical points, in this case the transition amplitude is exponentially suppressed. On the other hand, a consistent set of boundary data can be equivalently mapped to the set of lengths of the boundary triangulation. This set of lengths is a Dirichlet boundary condition for the Regge equations of motion to be used to determine the classical solution in the interior and evaluate the Hamilton function in (86).

The result (86) is what we expected from a theory of quantum gravity, and is the concrete realization of the equation (4) in the introduction.

CONCLUSIONS AND OUTLOOK

In this paper we have discussed a proposal for the semiclassical limit of Euclidean and Lorentzian spin-foams truncated to an arbitrary, finite triangulation (2-complex), where most calculations are done. We find (equation (86)) that the transition amplitudes yield the exponential of the Hamilton function of Regge-Einstein general relativity, as expected, up to \hbar -corrections and γ -corrections in the simultaneous semiclassical and continuum limit. The first corrections correspond to the standard WKB expansion of the path integral. The latter are new and are the effect of the discreteness of geometry, in the sense that the spectra of areas and volumes are discrete and the discreteness is controlled by the Immirzi parameter γ . The continuum limit we take is a looser concept of the ‘full’ continuum limit defined as the phase transition to a smooth spacetime manifold. However, working with a fixed, finite triangulation the only way of taking a continuum limit is to look at the continuous spectrum limit of the fundamental geometric operators. We have done this by letting γ run to zero, keeping fixed the macroscopic areas a_f (this is still a tentative proposal and must be further investigated). Remarkably, as explained in the paper, the result sheds new light on the previous calculation of the graviton propagator with the ‘new models’ [19], where the same kind of γ -corrections to the standard perturbative tree-level propagator have been found (one could speculate on potentially observable signatures of those pure LQG corrections).

It is also very interesting to remark that essentially the same continuum limit was considered by Bojowald [49] in the context of loop quantum cosmology. Quoting

¹⁰ Notice that also in the $\hbar \rightarrow 0$ limit with area fixed the spectrum becomes quasi-continuous, thus our separation of the semiclassical limit and the continuous spectrum limit is artificial and only a matter of speaking.

¹¹ The prefactors A and B are non-oscillatory and slowly varying.

its abstract: “*standard quantum cosmology is shown to be the simultaneous limit $\gamma \rightarrow 0$, $j \rightarrow \infty$ of loop quantum cosmology*”, a strong analogy it is worth studying further.

We did not study in great detail the physical effects of i) the contributions with inconsistent orientations, and ii) the degenerate triangulations. In principle, these do not pose a real problem: for example, see [50] on the fate of the sign-reversed exponentials in the bulk of the triangulation, and how a coherent boundary state peaked on the appropriate intrinsic *and* extrinsic curvature could be able to select the correct global orientation (see also [51]). In the Lorentzian setting, the issue of the wrong orientations has been intimately related to the issue of implementing causality in the spinfoam models [52–54].

Finally, we have also disregarded the potential ‘infrared’ divergencies associated to bubbles in the foam, similar to the loops of perturbative QFT, for which a suitable regularization and renormalization scheme [55–58] is required.

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